

Branching Ratio and Polarization of $B \rightarrow a_1(1260)(b_1(1235))\rho(\omega, \phi)$ Decays in the PQCD Approach

Zhi-Qing Zhang *

*Department of Physics, Henan University of Technology,
Zhengzhou, Henan 450052, P.R.China*

(Dated: March 1, 2013)

Abstract

Within the framework of perturbative QCD approach, we study the charmless two-body decays into final states involving one axial-vector (A), $a_1(1260)$ or $b_1(1235)$, and one vector (V), namely $\rho(\omega, \phi)$. Using the decays constants and the light-cone distribution amplitudes for these mesons derived from the QCD sum rule method, we find the following results: (a) Except the decays $\bar{B}^0 \rightarrow a_1^0 \rho^0(\omega)$, other tree-dominated decays $B \rightarrow a_1 \rho(\omega)$ have larger branching ratios, at the order of 10^{-5} . (b) Except the decays $\bar{B} \rightarrow b_1^+ \rho^-$ and $B^- \rightarrow b_1^0 \rho^-$, other $B \rightarrow b_1 \rho(\omega)$ decays have smaller branching ratios, at the order of 10^{-6} . (c) The decays $B \rightarrow a_1(b_1)\phi$ are highly suppressed and have very small branching ratios, at the order of 10^{-9} . (d) For the decays $\bar{B}^0 \rightarrow a_1^0 \rho^0$ and $B^- \rightarrow b_1^- \rho^0$, their two transverse polarizations are larger than their longitudinal polarizations, which are about 43.3% and 44.9%, respectively. (e) The two transverse polarizations have near values in the decays $B \rightarrow a_1 \rho(\omega)$, while have large differences in some of $B \rightarrow b_1 \rho(\omega)$ decays. (f) For the decays $B^- \rightarrow a_1^0 \rho^-, b_1^0 \rho^-$ and $\bar{B}^0 \rightarrow b_1^0 \rho^0, b_1^0 \omega$, where the transverse polarization fractions range from 4.7 to 7.5%, we calculate their direct CP-violating asymmetries with neglecting the transverse polarizations and find that those for two charged decays have smaller values, which are about 11.8% and -3.7% , respectively.

PACS numbers: 13.25.Hw, 12.38.Bx, 14.40.Nd

* Electronic address: zhangzhiqing@haut.edu.cn

I. INTRODUCTION

In general, the mesons are classified in J^{PC} multiplets. There are two types of orbitally excited axial-vector mesons, namely 1^{++} and 1^{+-} . The former includes $a_1(1260)$, $f_1(1285)$, $f_1(1420)$ and K_{1A} , which compose the 3P_1 -nonet, and the latter includes $b_1(1235)$, $h_1(1170)$, $h_1(1380)$ and K_{1B} , which compose the 1P_1 -nonet. There is an important character for these axial-vector mesons except $a_1(1260)$ and $b_1(1235)$, that is each different flavor state can mix with one another, which comes from the other nonet meson or the same nonet one. There is not mix between $a_1(1260)$ and $b_1(1235)$ because of the opposite C-parities. They do not also mix with others. So compared with other axial-vector mesons, these two mesons should have less uncertainties about their inner structures.

Like decay modes $B \rightarrow VV$, the charmless decays $B \rightarrow AV$ also have three polarization states and so are expected to have rich physics. In many $B \rightarrow VV$ decays, the informations on branching ratios and polarization fractions among various helicity amplitudes have been studied by many authors [1–4]. Through polarization studies, some underling helicity structures of the decay mechanism are proclaimed. They find that the polarization fractions follow the naive counting rule, that is $f_L \sim 1 - O(m_V^2/m_B^2)$, $f_{\parallel} \sim f_{\perp} \sim O(m_V^2/m_B^2)$. In the tree-dominated decay modes, such as $B^0 \rightarrow \rho^+\rho^-$, where the f_L is more than 90%. But if the contribution from the factorizable emission amplitudes is suppressed for some decay modes, this counting rule might be modified in some extent even dramatically by other contributions. For example, the polarization fractions of the decay $B \rightarrow \phi K^*$ are modified by its annihilation contribution. Whether the similar situation also occurs in the $B \rightarrow AV$ decay modes is worth researching by theories and experiments. We know that $a_1(1260)$ has some similar behaves with the vector meson, so one can expect that there should exist some similar characters in the branching ratios and the polarization fractions between decays $B \rightarrow a_1(1260)V$ and $B \rightarrow \rho V$, where $a_1(1260)$ is replaced by its scalar partner ρ . While it is not the case for $b_1(1235)$ because of its different characters in decay constant and light-cone distribution amplitude (LCDA) compared with those of $a_1(1260)$. For example, the longitude decay constant is very small for the charged $b_1(1235)$ states and vanishes under the SU(3) limit. It is zero for the neutral $b_1^0(1235)$ state. While the transverse decay constant of $a_1(1260)$ vanishes under the SU(3) limit. In the isospin limit, the chiral-odd (-even) LCDAs of meson $b_1(1235)$ are symmetric (antisymmetric) under the exchange of quark and anti-quark momentum fractions. It is just contrary to the symmetric behavior for $a_1(1260)$. In view of these differences, one can expect that there should exist very different results between $B \rightarrow a_1(1260)V$ and $B \rightarrow b_1(1235)V$. On the experimental side, a few of $B \rightarrow AV$ decays are studied, such as $B \rightarrow J/\psi K_1(1270)$ [5], $B^0 \rightarrow D^{*-}a_1^+$ [6], $B^0 \rightarrow a_1\rho$ [7], $B \rightarrow b_1\rho, b_1K^*$ [8]. In most of them only the upper limits for the branching ratios can be available. On the theoretical side, many charmless $B \rightarrow AV$ decays have been studied by Cheng and Yang in Ref. [9] where the branching ratios are very different with those calculated by naive factorization approach [10]. In most cases, the former are more large than the later. To clarify such large differences is another motivation of this work.

In the following, $a_1(1260)$ and $b_1(1235)$ are denoted as a_1 and b_1 in some places for convenience. The layout of this paper is as follows. In Sec.II, decay constants and light-cone distribution amplitudes of the relevant mesons are introduced. In Sec.III, we then analyze these decay channels using the PQCD approach. The numerical results and the

discussions are given in Sec. IV. The conclusions are presented in the final part.

II. DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES

For the wave function of the heavy B meson, we take

$$\Phi_B(x, b) = \frac{1}{\sqrt{2N_c}} (\not{P}_B + m_B) \gamma_5 \phi_B(x, b). \quad (1)$$

Here only the contribution of Lorentz structure $\phi_B(x, b)$ is taken into account, since the contribution of the second Lorentz structure $\bar{\phi}_B$ is numerically small [11] and has been neglected. For the distribution amplitude $\phi_B(x, b)$ in Eq.(1), we adopt the following model:

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_b^2} - \frac{1}{2}(\omega_b b)^2\right], \quad (2)$$

where ω_b is a free parameter, we take $\omega_b = 0.4 \pm 0.04$ GeV in numerical calculations, and $N_B = 91.745$ is the normalization factor for $\omega_b = 0.4$.

The wave function for the pseudoscalar meson P , such as $K, \pi, \eta^{(\prime)}$ meson is given as

$$\Phi_P(P, x, \zeta) \equiv \frac{1}{\sqrt{2N_c}} \gamma_5 [\not{P} \Phi^A(x) + m_0 \Phi^P(x) + \zeta m_0 (\not{v} \not{n} - v \cdot n) \Phi^T(x)]. \quad (3)$$

where P and x are the momentum and the momentum fraction of the pseudoscalar meson, respectively. The parameter ζ is either +1 or -1 depending on the assignment of the momentum fraction x .

In these decays, both the longitudinal and the transverse polarizations are involved for each final meson. For the vector mesons, their distribution amplitudes are defined as

$$\begin{aligned} \langle V(P, \epsilon_L^*) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle &= \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} [m_V \not{\epsilon}_L^* \phi_V(x) + \not{\epsilon}_L^* \not{P} \phi_V^t(x) + m_V \phi_V^s(x)]_{\alpha\beta}, \\ \langle V(P, \epsilon_T^*) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle &= \frac{1}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} [m_V \not{\epsilon}_T^* \phi_V^v(x) + \not{\epsilon}_T^* \not{P} \phi_V^T(x) \\ &\quad + m_V i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_T^{*v} n^\rho v^\sigma \phi_V^a(x)]_{\alpha\beta}, \end{aligned} \quad (4)$$

where $n(v)$ is the unit vector having the same (opposite) direction with the moving of the vector meson and x is the momentum fraction of q_2 quark. The distribution amplitudes of the axial-vectors have the same format as those of the vectors except the factor $i\gamma_5$ from the left hand:

$$\begin{aligned} \langle A(P, \epsilon_L^*) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle &= \frac{i\gamma_5}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} [m_A \not{\epsilon}_L^* \phi_A(x) + \not{\epsilon}_L^* \not{P} \phi_A^t(x) + m_A \phi_A^s(x)]_{\alpha\beta}, \\ \langle A(P, \epsilon_T^*) | \bar{q}_{2\beta}(z) q_{1\alpha}(0) | 0 \rangle &= \frac{i\gamma_5}{\sqrt{2N_c}} \int_0^1 dx e^{ixp \cdot z} [m_A \not{\epsilon}_T^* \phi_A^v(x) + \not{\epsilon}_T^* \not{P} \phi_A^T(x) \\ &\quad + m_A i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_T^{*v} n^\rho v^\sigma \phi_A^a(x)]_{\alpha\beta}. \end{aligned} \quad (5)$$

As for the upper twist-2 and twist-3 distribution functions of the final state mesons,

TABLE I: Decay constants and Gegenbauer moments for each meson (in MeV). The values are taken at $\mu = 1$ GeV.

f_ρ	f_ρ^T	f_ω	f_ω^T	f_ϕ	f_ϕ^T	f_{a_1}	$f_{b_1}^T$
209 ± 2	165 ± 9	195 ± 3	151 ± 9	231 ± 4	186 ± 9	238 ± 10	-180 ± 8
$a_2^\parallel(\rho, \omega)$	$a_2^\perp(\rho, \omega)$	$a_2^\parallel(\phi)$	$a_2^\perp(\phi)$	$a_2^\parallel(a_1(1260))$	$a_1^\perp(a_1(1260))$	$a_1^\parallel(b_1(1235))$	$a_2^\perp(b_1(1235))$
0.15 ± 0.07	0.14 ± 0.06	0.18 ± 0.08	0.14 ± 0.07	-0.02 ± 0.02	-1.04 ± 0.34	-1.95 ± 0.35	0.03 ± 0.19

$\phi_{V(A)}$, $\phi_{V(A)}^t$, $\phi_{V(A)}^s$, $\phi_{V(A)}^T$, $\phi_{V(A)}^v$ and $\phi_{V(A)}^a$ can be calculated by using light-cone QCD sum rule. We list the distribution functions of the vector (V) mesons, namely $\rho(\omega, \phi)$, as follows

$$\begin{cases} \phi_V(x) = \frac{f_V}{2\sqrt{2}N_c}\phi_\parallel(x), \phi_V^T(x) = \frac{f_V^T}{2\sqrt{2}N_c}\phi_\perp(x), \\ \phi_V^t(x) = \frac{f_V^T}{2\sqrt{2}N_c}h_\parallel^{(t)}(x), \phi_V^s(x) = \frac{f_V^T}{2\sqrt{4}N_c}\frac{d}{dx}h_\parallel^{(s)}(x), \\ \phi_V^v(x) = \frac{f_V}{2\sqrt{2}N_c}g_\perp^{(v)}(x), \phi_V^a(x) = \frac{f_V}{8\sqrt{2}N_c}\frac{d}{dx}g_\perp^{(a)}(x). \end{cases} \quad (6)$$

The axial-vector (A) mesons, here a_1 and b_1 , can be obtained by replacing all the ϕ_V with ϕ_A , by replacing $f_V^T(f_V)$ with f in Eq.(6). Here we use f to present both longitudinally and transversely polarized mesons $a_1(b_1)$ by assuming $f_{a_1}^T = f_{a_1} = f$ for a_1 and $f_{b_1}^T = f_{b_1} = f$ for b_1 . In Eq.(6), the twist-2 distribution functions are in the first line and can be expanded as

$$\phi_{\parallel,\perp} = 6x(1-x) \left[1 + a_2^{\parallel,\perp} \frac{3}{2}(5t^2 - 1) \right], \quad \text{for } V \text{ mesons}; \quad (7)$$

$$\phi_{\parallel,\perp} = 6x(1-x) \left[a_0^{\parallel,\perp} + 3a_1^{\parallel,\perp}t + a_2^{\parallel,\perp} \frac{3}{2}(5t^2 - 1) \right], \quad \text{for } A \text{ mesons}, \quad (8)$$

where the zeroth Gegenbauer moments $a_0^\perp(a_1) = a_0^\parallel(b_1) = 0$ and $a_0^\parallel(a_1) = a_0^\perp(b_1) = 1$.

As for twist-3 LCDAs, we use the asymptotic forms for V mesons:

$$\begin{aligned} h_\parallel^{(t)}(x) &= 3t^2, h_\parallel^{(s)}(x) = 6x(1-x), \\ g_\perp^{(a)}(x) &= 6x(1-x), g_\perp^{(v)}(x) = \frac{3}{4}(1+t^2). \end{aligned} \quad (9)$$

And we use the following forms for A mesons:

$$\begin{aligned} h_\parallel^{(t)}(x) &= 3a_0^\perp t^2 + \frac{3}{2}a_1^\perp t(3t^2 - 1), h_\parallel^{(s)}(x) = 6x(1-x)(a_0^\perp + a_1^\perp t), \\ g_\perp^{(a)}(x) &= 6x(1-x)(a_0^\parallel + a_1^\parallel t), g_\perp^{(v)}(x) = \frac{3}{4}a_0^\parallel(1+t^2) + \frac{3}{2}a_1^\parallel t^3. \end{aligned} \quad (10)$$

In Eqs.(7)-(10), the function $t = 2x - 1$. As in Ref.[12], the decay constants and the Gegenbauer moments $a_n^{\parallel,\perp}$ for each meson are quoted the numerical results [13–18] and listed in Table I.

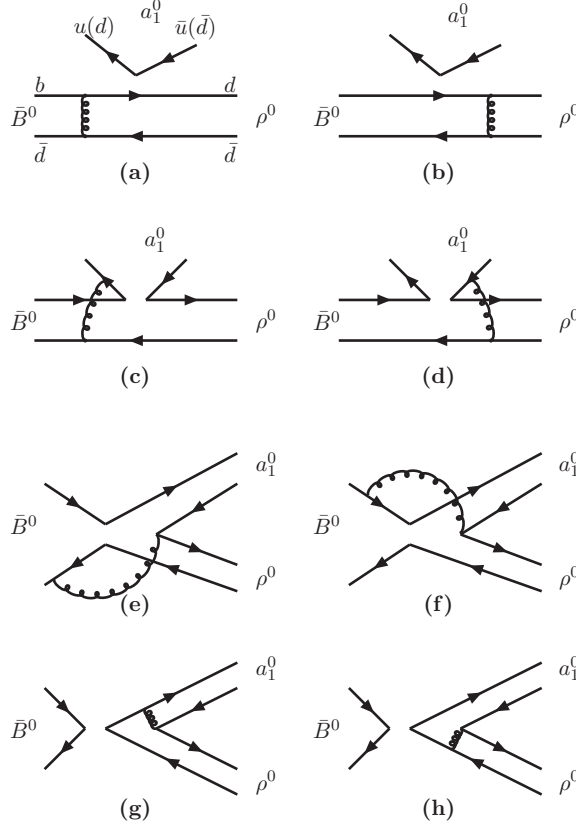


FIG. 1: Diagrams contributing to the decay $\bar{B}^0 \rightarrow a_1^0 \rho^0$.

III. THE PERTURBATIVE QCD CALCULATION

The PQCD approach is an effective theory to handle hadronic B decays [19–21]. Because it takes into account the transverse momentum of the valence quarks in the hadrons, one will encounter double logarithm divergences when the soft and the collinear momenta overlap. Fortunately, these large double logarithm can be re-summed into the Sudakov factor [22]. There are also another type of double logarithms which arise from the loop corrections to the weak decay vertex. These double logarithms can also be re-summed and resulted in the threshold factor [23]. This factor decreases faster than any other power of the momentum fraction in the threshold region, which removes the endpoint singularity. This factor is often parameterized into a simple form which is independent on channels, twists and flavors [24]. Certainly, when the higher order diagrams only suffer from soft or collinear infrared divergence, it is ease to cure by using the eikonal approximation [25]. Controlling these kinds of divergences reasonably makes the PQCD approach more self-consistent.

Here we take the decay $\bar{B}^0 \rightarrow a_1^0 \rho^0$ as an example, whose part of diagrams are shown in Figure 1. These eight Feynman diagrams belong to the condition of a_1^0 meson being at the emission position. Another eight Feynman diagrams obtained by exchanging the positions of a_1^0 and ρ^0 in Fig.1 also contribute to the decay. All of these single hard gluon exchange diagrams contain all of the leading order contributions to $\bar{B}^0 \rightarrow a_1^0 \rho^0$ in the PQCD approach. Similar to the $B \rightarrow VV$ decay modes, such as $B \rightarrow \rho\rho$ [1] and

$B \rightarrow K^* \rho(\omega)$ [2], both longitudinal and transverse polarizations can contribute to the decay width. So we can get three kinds of polarization amplitudes M_L (longitudinal) and $M_{N,T}$ (transverse) by calculating these diagrams. Because of the aforementioned distribution amplitudes of the axial-vectors having the same format as those of the vectors except a factor, so the formulas of here considered decays can be obtained from the ones of $B \rightarrow VV$ decays by some replacements. Certainly, there also exists a difference: if the emitted meson is b_1 for the factorizable emission diagrams, the amplitudes contributed by the $(V-A)(V \pm A)$ operators would be zero due to the vanishing decay constant f_{b_1} . For the tree-dominated decays, the contributions from the factorizable emission diagrams, namely Fig.(a),(b), are very important. In the PQCD approach, the form factor can be extracted from the amplitudes obtained by calculating such diagrams, where the two transverse amplitudes are highly suppressed by the factor $r_{a_1(b_1)} \cdot r_{\rho(\omega)}$ compared with the longitudinal amplitudes. Here $r_{a_1(b_1)} = \frac{m_{a_1(b_1)}}{m_B}$ and $r_{\rho(\omega)} = \frac{m_{\rho(\omega)}}{m_B}$. To some decays, the non-factorizable emission diagrams, namely Fig.(c),(d), play an more important role, where the contributions from the transverse polarizations are not suppressed. Certainly, the contributions from the non-factorizable and the factorizable annihilation diagrams, that are Fig.(g),(h) and Fig.(e),(f), can also not be neglected.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We use the following input parameters in the numerical calculations [26, 27]:

$$f_B = 190 \text{ MeV}, M_B = 5.28 \text{ GeV}, M_W = 80.41 \text{ GeV}, \quad (11)$$

$$\tau_{B^\pm} = 1.638 \times 10^{-12} \text{ s}, \tau_{B^0} = 1.525 \times 10^{-12} \text{ s}, \quad (12)$$

$$|V_{ud}| = 0.974, |V_{td}| = 8.58 \times 10^{-3}, \alpha = (91.0 \pm 3.9)^\circ, \quad (13)$$

$$|V_{ub}| = 3.54 \times 10^{-3}, |V_{tb}| = 0.999. \quad (14)$$

In the B-rest frame, the decay rates of $B \rightarrow a_1(b_1)V$, where V represents ρ, ω, ϕ , can be written as

$$\Gamma = \frac{G_F^2 (1 - r_{a_1(b_1)}^2)}{32\pi M_B} \sum_{\sigma=L,N,T} \mathcal{M}^{\sigma\dagger} \mathcal{M}^\sigma, \quad (15)$$

where \mathcal{M}^σ is the total decay amplitude of each considered decay. The subscript σ is the helicity states of the two final mesons with one longitudinal component and two transverse ones. The decay amplitude can be decomposed into three scalar amplitudes a, b, c according to

$$\begin{aligned} \mathcal{M}^\sigma &= \epsilon_{2\mu}^*(\sigma) \epsilon_{3\nu}^*(\sigma) \left[a g^{\mu\nu} + \frac{b}{M_2 M_3} P_B^\mu P_B^\nu + i \frac{c}{M_2 M_3} \epsilon^{\mu\nu\alpha\beta} P_{2\alpha} P_{3\beta} \right] \\ &= \mathcal{M}_L + \mathcal{M}_N \epsilon_2^*(\sigma = T) \cdot \epsilon_3^*(\sigma = T) + i \frac{\mathcal{M}_T}{M_B^2} \epsilon^{\alpha\beta\gamma\rho} \epsilon_{2\alpha}^*(\sigma) \epsilon_{3\beta}^*(\sigma) P_{2\gamma} P_{3\rho}, \end{aligned} \quad (16)$$

where M_2 and M_3 are the masses of the two final mesons $a_1(b_1)$ and $\rho(\omega, \phi)$, respectively.

The amplitudes $\mathcal{M}_L, \mathcal{M}_N, \mathcal{M}_T$ can be expressed as

$$\begin{aligned}\mathcal{M}_L &= a \epsilon_2^*(L) \cdot \epsilon_3^*(L) + \frac{b}{M_2 M_3} \epsilon_2^*(L) \cdot P_3 \epsilon_3^*(L) \cdot P_2, \\ \mathcal{M}_N &= a, \quad \mathcal{M}_T = \frac{M_B^2}{M_2 M_3} c.\end{aligned}\tag{17}$$

We can use the amplitudes with different Lorentz structures to define the helicity amplitudes, one longitudinal amplitudes H_0 and two transverse amplitudes H_{\pm} :

$$H_0 = M_B^2 \mathcal{M}_L, \quad H_{\pm} = M_B^2 \mathcal{M}_N \mp M_2 M_3 \sqrt{r^2 - 1} \mathcal{M}_T,\tag{18}$$

where the ratio $r = P_2 \cdot P_3 / (M_2 M_3)$. After the helicity summation, we can get the relation

$$\sum_{\sigma=L,N,T} \mathcal{M}^{\sigma\dagger} \mathcal{M}^{\sigma} = |\mathcal{M}_L|^2 + 2(|\mathcal{M}_N|^2 + |\mathcal{M}_T|^2) = |H_0|^2 + |H_+|^2 + |H_-|^2.\tag{19}$$

Certainly another equivalent set of helicity amplitudes are often used, that is

$$\begin{aligned}A_0 &= -M_B^2 \mathcal{M}_L, \\ A_{\parallel} &= \sqrt{2} M_B^2 \mathcal{M}_N, \\ A_{\perp} &= M_2 M_3 \sqrt{2(r^2 - 1)} \mathcal{M}_T.\end{aligned}\tag{20}$$

Using this set of helicity amplitudes, we can define three polarization fractions $f_{0,\parallel,\perp}$:

$$f_{0,\parallel,\perp} = \frac{|A_{0,\parallel,\perp}|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}.\tag{21}$$

The matrix elements \mathcal{M}_j of the operators in the weak Hamiltonian can be calculated by using PQCD approach, which are written as as

$$\begin{aligned}M_j &= V_{ub} V_{ud}^* T_j - V_{tb} V_{td}^* P_j \\ &= V_{ub} V_{ud}^* T_j (1 + z_j e^{i(\alpha + \delta_j)}),\end{aligned}\tag{22}$$

where $j = L, N, T$ and α is the Cabibbo-Kobayashi-Maskawa weak phase angle, defined via $\alpha = \arg[-\frac{V_{td} V_{tb}^*}{V_{ud} V_{ub}^*}]$. Here we leave this angle as a free parameter. δ_j is the relative strong phase between the tree and the penguin amplitudes, which are denoted as " T_j " and " P_j ", respectively. The term z_j describes the ratio of penguin to tree contributions and is defined as

$$z_j = \left| \frac{V_{tb} V_{td}^*}{V_{ub} V_{ud}^*} \right| \left| \frac{P_j}{T_j} \right|.\tag{23}$$

In the same way, it is easy to write decay amplitude $\overline{\mathcal{M}}_j$ for the corresponding conjugated decay mode:

$$\begin{aligned}\overline{\mathcal{M}}_j &= V_{ub}^* V_{ud} T_j - V_{tb}^* V_{td} P_j \\ &= V_{ub}^* V_{ud} T_j (1 + z_j e^{i(-\alpha + \delta_j)}).\end{aligned}\tag{24}$$

TABLE II: Branching ratios (in units of 10^{-6}) for the decays $B \rightarrow a_1(1260)\rho(\omega, \phi)$ and $B \rightarrow b_1(1235)\rho(\omega, \phi)$. In our results, the errors for these entries correspond to the uncertainties from ω_B and threshold resummation parameter c , respectively. For comparison, we also listed the results predicted by QCDF approach [9] and naive factorization approach [10].

	This work	[9]	[10]
$\bar{B}^0 \rightarrow a_1^+ \rho^-$	$33.6^{+9.9+15.8}_{-7.4-15.8}$	$23.9^{+10.5+3.2}_{-9.2-0.4}$	4.3
$\bar{B}^0 \rightarrow a_1^- \rho^+$	$27.1^{+8.0+9.2}_{-6.0-9.2}$	$36.0^{+3.5+3.5}_{-4.0-0.7}$	4.7
$\bar{B}^0 \rightarrow a_1^0 \rho^0$	$0.64^{+0.12+0.04}_{-0.10-0.04}$	$1.2^{+2.0+5.1}_{-0.7-0.3}$	0.01
$B^- \rightarrow a_1^0 \rho^-$	$27.7^{+7.8+7.9}_{-5.9-7.9}$	$17.8^{+10.1+3.1}_{-6.4-0.2}$	2.4
$B^- \rightarrow a_1^- \rho^0$	$21.9^{+5.9+9.3}_{-4.6-9.3}$	$23.2^{+3.6+4.8}_{-2.9-0.1}$	3.0
$\bar{B}^0 \rightarrow a_1^0 \omega$	$0.83^{+0.27+0.40}_{-0.20-0.40}$	$0.2^{+0.1+0.4}_{-0.1-0.0}$	0.003
$B^- \rightarrow a_1^- \omega$	$14.4^{+4.8+6.0}_{-3.5-6.0}$	$22.5^{+3.4+3.0}_{-2.7-0.7}$	2.2
$\bar{B}^0 \rightarrow a_1^0 \phi$	$0.0029^{+0.0007+0.0006}_{-0.0006-0.0006}$	$0.002^{+0.002+0.009}_{-0.001-0.000}$	0.0005
$B^- \rightarrow a_1^- \phi$	$0.0058^{+0.0015+0.0013}_{-0.0013-0.0013}$	$0.01^{+0.01+0.04}_{-0.00-0.00}$	0.001
$\bar{B}^0 \rightarrow b_1^+ \rho^-$	$46.8^{+15.6+19.1}_{-11.3-19.1}$	$32.1^{+16.5+12.0}_{-14.7-4.6}$	1.6
$\bar{B}^0 \rightarrow b_1^- \rho^+$	$2.2^{+0.3+0.1}_{-0.3-0.1}$	$0.6^{+0.6+1.9}_{-0.3-0.2}$	0.55
$\bar{B}^0 \rightarrow b_1^0 \rho^0$	$3.4^{+0.4+0.4}_{-0.5-0.4}$	$3.2^{+5.2+1.7}_{-2.0-0.4}$	0.002
$B^- \rightarrow b_1^0 \rho^-$	$22.9^{+8.7+24.3}_{-6.3-24.3}$	$29.1^{+16.2+5.4}_{-10.6-5.9}$	0.86
$B^- \rightarrow b_1^- \rho^0$	$1.4^{+0.2+0.3}_{-0.2-0.3}$	$0.9^{+1.7+2.6}_{-0.6-0.5}$	0.36
$\bar{B}^0 \rightarrow b_1^0 \omega$	$2.8^{+0.7+0.2}_{-0.6-0.2}$	$0.1^{+0.2+1.6}_{-0.0-0.0}$	0.004
$B^- \rightarrow b_1^- \omega$	$2.1^{+0.4+0.7}_{-0.2-0.7}$	$0.8^{+1.4+3.1}_{-0.5-0.3}$	0.38
$\bar{B}^0 \rightarrow b_1^0 \phi$	$0.003^{+0.001+0.000}_{-0.001-0.000}$	$0.01^{+0.01+0.01}_{-0.00-0.00}$	0.0002
$B^- \rightarrow b_1^- \phi$	$0.006^{+0.003+0.001}_{-0.002-0.001}$	$0.02^{+0.02+0.03}_{-0.01-0.00}$	0.0004

So the CP-averaged branching ratio for each considered decay is defined as

$$\mathcal{B} = (|\mathcal{M}_j|^2 + |\overline{\mathcal{M}}_j|^2)/2 = |V_{ub}V_{ud}^*|^2 \left[T_L^2(1 + 2z_L \cos \alpha \cos \delta_L + z_L^2) + 2 \sum_{j=N,T} T_j^2(1 + 2z_j \cos \alpha \cos \delta_j + z_j^2) \right]. \quad (25)$$

Like the decays of B to two vector mesons, there are also 3 types of helicity amplitudes, so corresponding to 3 types of z_j and δ_j , respectively. It is easy to see that the dependence of decay width on δ and α is more complicated compared with that for the decays of B to pseudoscalar mesons.

Using the input parameters and the wave functions as specified in this section and Sec.II, it is easy to get the branching ratios for the considered decays which are listed in Table II, where the first error comes from the uncertainty in the B meson shape parameter $\omega_b = 0.40 \pm 0.04$ GeV, the second one is from the threshold resummation parameter c , and it varies from 0.3 to 0.4. In Fig.2 and Fig.3, we also show the Cabibbo-Kobayashi-Maskawa angle α dependence of the branching ratios of decays $B \rightarrow a_1 \rho(\omega)$ and $B \rightarrow b_1 \rho(\omega)$.

From Table II, one can find that except decays $\bar{B}^0 \rightarrow a_1^0 \rho^0, a_1^0 \omega$, the branching ratios

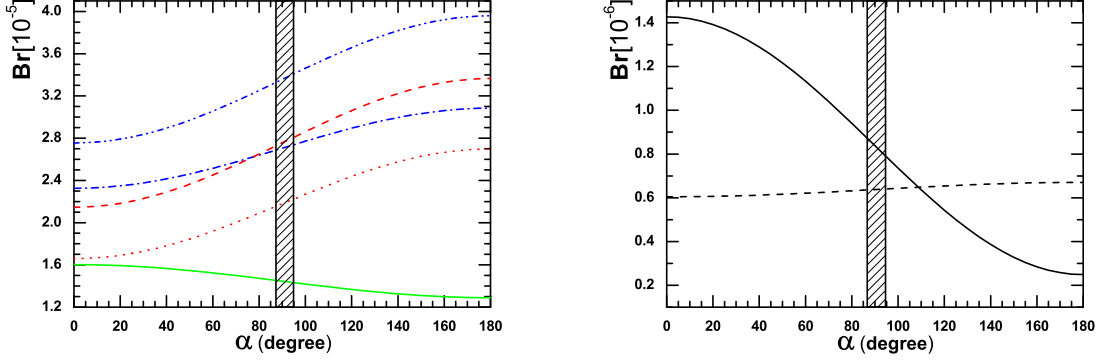


FIG. 2: The dependence of the branching ratios on the Cabibbo-Kobayashi-Maskawa angle α . In the left panel, the solid line is for $B^- \rightarrow a_1^- \omega$, dotted line for $B^- \rightarrow a_1^- \rho^0$, dot-dashed line for $\bar{B}^0 \rightarrow a_1^- \rho^+$, dashed line for $B^- \rightarrow b_1^0 \rho^-$, dot-dot-dashed line for $\bar{B}^0 \rightarrow a_1^+ \rho^-$. In the right panel, the solid line is for $\bar{B}^0 \rightarrow a_1^0 \omega$ and the dashed line is for $\bar{B}^0 \rightarrow a_1^0 \rho^0$. The vertical band shows the range of α : 91.0 ± 3.9 .

of other tree-dominated decays $B \rightarrow a_1 \rho(\omega)$ are all at the order of 10^{-5} . Most of the contributions to such larger branching ratios are from the factorizable emission diagrams (a) and (b), which contribute to the $B \rightarrow \rho(\omega)$ ($B \rightarrow a_1$) form factors. Because of the large Wilson coefficients $C_2 + C_1/3$ in the amplitudes contributed by the tree operators O_1 and O_2 , the branch ratios are almost proportionate to the corresponding form factors. Certainly, they are also related to the decay constants f_{a_1} ($f_{\rho,\omega}$). As the basic input values, they are the same in many factorization approaches, for example, PQCD and QCDF approaches. While for the form factors, there exist some differences between these two approaches. For QCDF approach, the form factors are used as the input values, which are obtained from light-cone sum rules. In Ref. [9], the form factors $A_0^{B\rho}$ and $V_0^{Ba_1}$ are both about 0.30, and $V_0^{Bb_1}$ is about -0.39 , where the authors put an additional minus sign by taking the convention of the decay constants of a_1 and b_1 being of the same sign. In this convention, the corresponding form factors have opposite signs. For the PQCD approach, the form factors can be calculated perturbatively. From our calculations, we find that the values of $A_0^{B\rho}$, $V_0^{Ba_1}$ and $V_0^{Bb_1}$ are about 0.25, 0.33 and 0.44, respectively. If the decay is governed by the form factor $A_0^{B\rho}$, its branching ratio predicted by PQCD approach would be smaller than that obtained by QCDF approach, for example, $\bar{B}^0 \rightarrow a_1^- \rho^+$. On the contrary, if the decay is governed by the form factor $V_0^{Ba_1}$, the result for the PQCD approach would have a larger value, the decay $\bar{B}^0 \rightarrow a_1^+ \rho^-$ is in this case. So to accurately determine these form factors is very important. The branching ratio of $B^- \rightarrow a_1^- \rho^0$ is larger than that of $B^- \rightarrow a_1^- \omega$, one reason is that the form factor $A_0^{B\rho}$ is a little larger than $A_0^{B\omega}$, which is about 0.23. The other reason is the different interferences from $d\bar{d}$ and $u\bar{u}$: constructive interference between $-d\bar{d}$ and $u\bar{u}$ which compose ρ , destructive interference between $d\bar{d}$ and $u\bar{u}$ which compose ω . But there is a contrary situation for the QCDF approach between these two decays. Although the neutral decays $\bar{B}^0 \rightarrow a_1^0 \rho^0, a_1^0 \omega$ are also

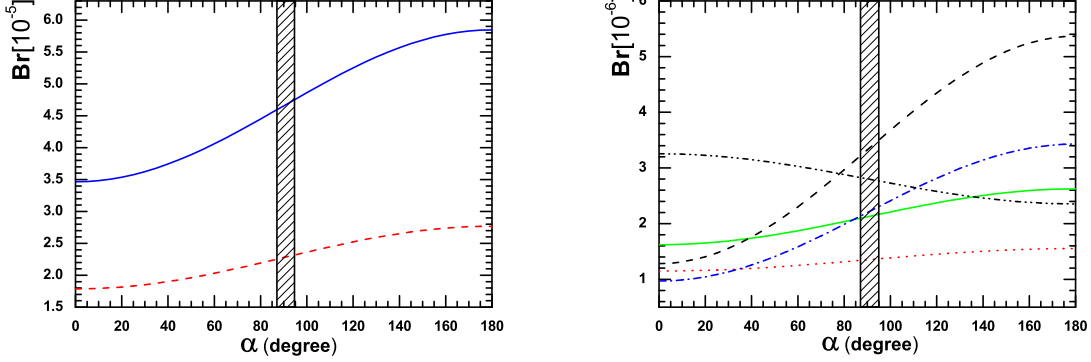


FIG. 3: The dependence of the branching ratios on the Cabibbo-Kobayashi-Maskawa angle α . In the left panel, the solid line is for $\bar{B}^0 \rightarrow b_1^+ \rho^-$ and the dashed line is for $B^- \rightarrow b_1^0 \rho^-$. In the right panel, the dotted line is for $B^- \rightarrow b_1^- \rho^0$, solid line for $B^- \rightarrow b_1^- \omega$, dot-dashed line for $\bar{B}^0 \rightarrow b_1^- \rho^+$, dot-dot-dashed line for $\bar{B}^0 \rightarrow b_1^0 \omega$ and dashed line for $\bar{B}^0 \rightarrow b_1^0 \rho^0$. The vertical band shows the range of α : 91.0 ± 3.9 .

TABLE III: Polarization amplitudes of different diagrams for the decays $\bar{B}^0 \rightarrow a_1^+ \rho^-$, $a_1^0 \rho^0$ ($\times 10^{-2} \text{GeV}^3$).

Decay mode	Pol. amp.	(a) and (b)	(c) and (d)	(e) and (f)	(g) and (h)
$a_1^+ \rho^-$	$A(T_L)$	-219.2	$8.1 - 3.8i$	$-1.2 + 9.0i$	$-0.5 - 0.1i$
	$A(T_N)$	22.8	$-7.1 + 5.2i$	$-0.2 - 0.2ii$	$0.6 + 0.03i$
	$A(T_T)$	-57.3	$-12.9 + 3.2i$	$0.1 - 0.4i$	$-1.0 - 0.2i$
	$A(P_L)$	8.8	$-0.09 + 0.13i$	$0.59 + 1.7i$	$-1.7 - 3.4i$
	$A(P_N)$	0.9	$0.26 - 0.17i$	$-0.03 - 0.01i$	$0.7 + 3.4i$
	$A(P_T)$	2.2	$0.49 - 0.05i$	$-0.03 + 0.01i$	$1.1 + 6.6i$
$a_1^0 \rho^0$	$A(T_L)$	-5.7	$18.4 - 7.3i$	$1.1 - 4.7i$	$1.4 - 1.0i$
	$A(T_N)$	0.5	$-11.0 + 6.5i$	$0.06 + 0.06i$	$0.54 + 0.05i$
	$A(T_T)$	-0.1	$-20.8 + 5.2i$	$0.00 + 0.17i$	$-1.1 - 0.15i$
	$A(P_L)$	0.8	$0.36 + 0.12i$	$0.33 + 1.24i$	$-0.12 - 0.06i$
	$A(P_N)$	-0.15	$0.26 - 0.15i$	$-0.05 - 0.02i$	$-0.08 + 0.04i$
	$A(P_T)$	-0.06	$0.50 - 0.1i$	$-0.08 + 0.00i$	$-0.34 - 0.14i$

tree dominant, their tree operator contributions are highly suppressed compared with the two charged decays $B^- \rightarrow a_1^- \rho^0$, $a_1^- \omega$ (shown in Table III). So their branching ratios are small and at the order of 10^{-7} . Certainly, we only give the leading order results and they might like decays $B \rightarrow \rho^0 \rho^0$, $\rho^0 \omega$, which are sensitive to the next leading order contributions.

As to the tree-dominated decays $B \rightarrow b_1^+ \rho^-, b_1^0 \rho^-$, which are governed by the decay constant f_ρ and the form factor of $B \rightarrow b_1$, they also have large branching ratios. Although $B \rightarrow b_1^- \rho^+$ is color allowed decay, its branching ratio is highly suppressed due to the decay constant f_{b_1} being very small and vanishing under the isospin limit. One should admit that each amplitude for the decays $B^- \rightarrow b_1^-(b_1^0) \rho^0$ has near value in magnitude with the corresponding one for the decays $B^- \rightarrow b_1^-(b_1^0) \omega$, but the sign differences before $d\bar{d}$ in the mesons ρ and ω will induce some discrepancies in the branching ratios. Like the decays $B \rightarrow \pi\phi, a_0(1450)\phi$ [28, 29], whose branching ratios are at the order of $10^{-8} \sim 10^{-9}$, the decays $B \rightarrow a_1(b_1)\phi$ are induced by the flavor-changing neutral current (FCNC) interactions and highly suppressed by the small Wilson coefficients for penguin operators. Moreover, there is no the contribution from the annihilation diagram. So one expects that their branching ratios are also very small.

From Table II, One can find that our predictions are well consistent with the results calculated by QCDF approach for most decays. Certainly, there also exist large differences for some decays, which are needed to clarify by the present LHCb experiments. At the present, BaBar has given the upper limits of the branching ratios for the decays $B \rightarrow b_1 \rho$, ranging from $1.4 \sim 5.2 \times 10^{-6}$ at the 90% confidence level [8], which are not far away from our predictions for the decays $\bar{B}^0 \rightarrow b_1^0 \rho^0$ and $B^- \rightarrow b_1^- \rho^0$, but much smaller than those of $\bar{B}^0 \rightarrow b_1^+ \rho^-$ and $B^- \rightarrow b_1^0 \rho^-$. In Ref.[7], the BarBar collaboration searched the decay $\bar{B}^0 \rightarrow a_1^\pm \rho^\mp$ and obtained an upper limit of 61×10^{-6} by assuming that a_1^\pm decays exclusively to $\rho^0 \pi^\pm$. Our prediction for the branching ratio of $\bar{B}^0 \rightarrow a_1^\pm \rho^\mp$ is about 60×10^{-6} , which agrees with the experiment.

In Table IV, we list the polarization fractions of $B \rightarrow a_1 \rho(\omega), b_1 \rho(\omega)$ decays and find that the longitudinal polarizations are dominant in most of these decays, which occupy more than 80%. For the tree-dominated decays, the main contributions come from the factorizable emission diagrams, where the two kinds of transverse polarization amplitudes are highly suppressed by the aforementioned factor $r_{a_1(b_1)} \cdot r_{\rho(\omega)}$. From Table IV, One can find that f_\parallel and f_\perp have near values and both about a few percent in general. Certainly, for the decays $\bar{B}^0 \rightarrow a_1^0 \rho^0$ and $B^- \rightarrow b_1^- \rho^0(\omega)$, their polarization fractions are very different with those of other decays. In the decay $\bar{B}^0 \rightarrow a_1^0 \rho^0$, the contributions from the two transverse polarization components become prominent and are larger than that from the longitudinal component. It is because that the decay is suppressed by the cancelation of Wilson coefficients $C_1 + C_2/3$ for the color-suppressed amplitude. So the contribution from the factorizable emission diagrams become very small. The left dominant contributions are the non-factorizable amplitudes from tree operators, where either of the transverse polarizations is not suppressed compared with the longitudinal polarization. Therefore numerically we get a small longitudinal polarization fraction of about 43%. In Table V, if we ignore the contribution from the non-factorizable amplitudes of $\bar{B}^0 \rightarrow a_1^0 \rho^0$ and find that the longitudinal polarization becomes dominant, but the branching ratio becomes very small. If we ignore the contributions from its penguin operators or annihilation diagrams, the results have small changes. As to the other charged decays $B^- \rightarrow b_1^- \rho^0(\omega)$, either of their transverse polarizations is very sensitive to the contributions listed in lines (2)-(4) in Table V.

Now we turn to the evaluations of the CP-violating asymmetries in PQCD approach. The CP asymmetries of $B^0/\bar{B}^0 \rightarrow a_1^\pm(b_1^\pm) \rho^\mp$ are very complicated and left for future study. Here we only research the decays $B^- \rightarrow a_1^0(b_1^0) \rho^-$ and $\bar{B}^0 \rightarrow b_1^0 \rho^0(\omega)$, where the transverse polarization fractions are very small and range from 4.7 to 7.5%. Using Eq.(22)

TABLE IV: Longitudinal polarization fraction (f_L) and two transverse polarization fractions (f_{\parallel} , f_{\perp}) for the decays $B \rightarrow a_1(1260)\rho(\omega)$ and $B \rightarrow b_1(1235)\rho(\omega)$. In our results, the uncertainties of f_L come from ω_B and threshold resummation parameter c . The results of f_L predicted by the QCDF approach are also displayed in parentheses for comparison.

	$f_L(\%)$	$f_{\parallel}(\%)$	$f_{\perp}(\%)$
$\bar{B}^0 \rightarrow a_1^+ \rho^-$	$90.7^{+0.2+1.3}_{-0.2-1.3} (82^{+5}_{-13})$	3.9	5.4
$\bar{B}^0 \rightarrow a_1^- \rho^+$	$90.4^{+0.0+0.1}_{-0.1-0.1} (84^{+2}_{-6})$	5.2	4.4
$\bar{B}^0 \rightarrow a_1^0 \rho^0$	$43.3^{+1.2+2.9}_{-1.3-2.9} (82^{+6}_{-68})$	29.7	27.0
$B^- \rightarrow a_1^0 \rho^-$	$93.6^{+0.2+0.1}_{-0.2-0.1} (91^{+3}_{-10})$	2.8	3.6
$B^- \rightarrow a_1^- \rho^0$	$82.3^{+0.1+2.0}_{-0.3-2.0} (89^{+11}_{-18})$	9.3	8.4
$\bar{B}^0 \rightarrow a_1^0 \omega$	$80.7^{+0.3+3.4}_{-0.1-3.4} (75^{+11}_{-65})$	9.9	9.4
$B^- \rightarrow a_1^- \omega$	$79.5^{+0.6+2.2}_{-0.6-2.2} (88^{+10}_{-14})$	8.9	11.6
$\bar{B}^0 \rightarrow b_1^+ \rho^-$	$95.4^{+0.2+0.1}_{-0.1-0.1} (96^{+1}_{-2})$	2.2	2.4
$\bar{B}^0 \rightarrow b_1^- \rho^+$	$95.8^{+0.5+1.1}_{-0.5-1.1} (98^{+0}_{-33})$	1.7	2.5
$\bar{B}^0 \rightarrow b_1^0 \rho^0$	$95.3^{+0.2+0.4}_{-0.4-0.4} (99^{+0}_{-18})$	2.8	1.9
$B^- \rightarrow b_1^0 \rho^-$	$92.5^{+0.9+0.6}_{-1.1-0.6} (96^{+1}_{-6})$	0.8	6.7
$B^- \rightarrow b_1^- \rho^0$	$44.9^{+1.8+5.6}_{-2.0-5.6} (90^{+5}_{-38})$	1.1	54.0
$\bar{B}^0 \rightarrow b_1^0 \omega$	$93.5^{+0.2+0.3}_{-0.1-0.3} (4^{+96}_{-0})$	4.3	2.2
$B^- \rightarrow b_1^- \omega$	$73.1^{+0.5+1.0}_{-0.6-1.0} (91^{+7}_{-33})$	25.5	1.4

and Eq.(24), one can get the expression for the direct CP-violating asymmetry:

$$\mathcal{A}_{CP}^{dir} = \frac{|\overline{\mathcal{M}}|^2 - |\mathcal{M}|^2}{|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2} = \frac{2z_L \sin \alpha \sin \delta_L}{(1 + 2z_L \cos \alpha \cos \delta_L + z_L^2)}. \quad (26)$$

Here for our considered four decays, the contributions from the transverse polarizations are very small, so we neglected them in our calculations. Using the input parameters and the wave functions as specified in this section and Sec.II, one can find the PQCD predictions (in units of 10^{-2}) for the direct CP-violating asymmetries of the considered decays:

$$\mathcal{A}_{CP}^{dir}(B^- \rightarrow a_1^0 \rho^-) = 11.8^{+1.6+0.0}_{-1.4-0.0}, \quad (27)$$

$$\mathcal{A}_{CP}^{dir}(B^- \rightarrow b_1^0 \rho^-) = -3.7^{+0.4+1.2}_{-0.3-1.2}, \quad (28)$$

$$\mathcal{A}_{CP}^{dir}(\bar{B}^0 \rightarrow b_1^0 \rho^0) = 23.8^{+4.3+1.9}_{-4.2-1.9}, \quad (29)$$

$$\mathcal{A}_{CP}^{dir}(\bar{B}^0 \rightarrow b_1^0 \omega) = 80.3^{+3.8+3.2}_{-4.8-3.2}, \quad (30)$$

where the errors are induced by the uncertainties of B meson shape parameter $\omega_b = 0.4 \pm 0.04$ and the threshold resummation parameter c , varying from 0.3 to 0.4.

V. CONCLUSION

In this paper, by using the decay constants and the light-cone distribution amplitudes derived from QCD sum-rule method, we research $B \rightarrow a_1(1260)\rho(\omega, \phi), b_1(1235)\rho(\omega, \phi)$

TABLE V: Contributions from different parts in the decays $\bar{B}^0 \rightarrow a_1^0 \rho^0$ and $B^- \rightarrow b_1^- \omega$: line (1) is for full contribution, line (2), (3) and (4) are the contributions after ignoring annihilation diagrams, penguin operators and non-factorization diagrams, respectively.

$\bar{B}^0 \rightarrow a_1^0 \rho^0$	$\text{Br}(10^{-7})$	$f_L(\%)$	$f_{\parallel}(\%)$	$f_{\perp}(\%)$
(1)	6.4	43.3	29.7	27.0
(2)	5.1	28.4	40.4	31.2
(3)	6.3	42.5	30.1	27.4
(4)	0.2	86.1	9.4	4.5
$B^- \rightarrow b_1^- \omega$	$\text{Br}(10^{-6})$	$f_L(\%)$	$f_{\parallel}(\%)$	$f_{\perp}(\%)$
(1)	2.1	73.1	25.5	1.4
(2)	0.9	63.5	18.5	18.0
(3)	0.7	67.9	0.1	32.0
(4)	1.8	83.2	11.1	5.7

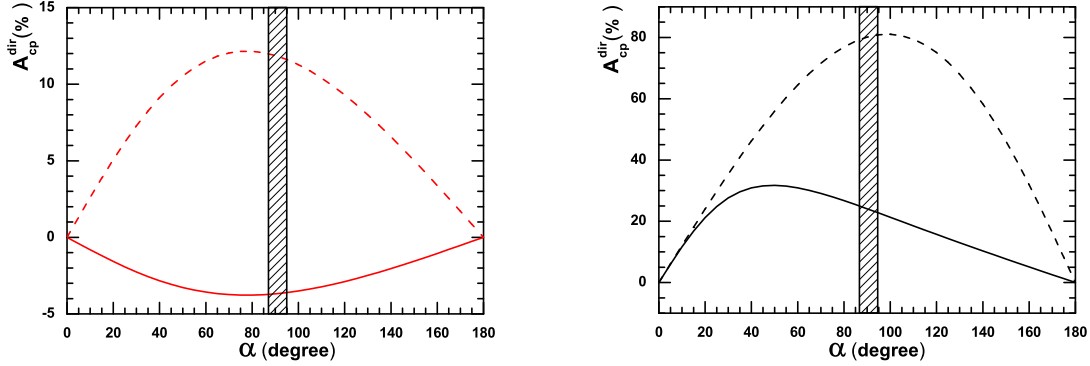


FIG. 4: Direct CP-violating asymmetry as a function of Cabibbo-Kobayashi-Maskawa angle α . In the left panel, the solid line is for $B^- \rightarrow b_1^0 \rho^-$ and the dashed line is for $B^- \rightarrow a_1^0 \rho^-$. In the right panel, the solid line is for $\bar{B}^0 \rightarrow b_1^0 \rho^0$ and the dashed line is for $\bar{B}^0 \rightarrow b_1^0 \omega$. The vertical band shows the range of α : 91.0 ± 3.9 .

decays in PQCD factorization approach and find that

- Except the decays $\bar{B}^0 \rightarrow a_1^0 \rho^0(\omega)$, other tree-dominated $B \rightarrow a_1 \rho(\omega)$ decays have larger branching ratios, at the order of 10^{-5} . Except the decays $\bar{B} \rightarrow b_1^+ \rho^-$ and $B^- \rightarrow b_1^0 \rho^-$, other $B \rightarrow b_1 \rho(\omega)$ decays have smaller branching ratios, at the order of 10^{-6} . The decays $B \rightarrow a_1(b_1)\phi$ are highly suppressed and have very small branching ratios, at the order of 10^{-9} .
- For the decays $\bar{B}^0 \rightarrow a_1^0 \rho^0$ and $B^- \rightarrow b_1^- \rho^0$, their two transverse polarizations

are larger than their longitudinal polarizations, which are about 43.3% and 44.9%, respectively. The two transverse polarization fractions have near values in the decays $B \rightarrow a_1 \rho(\omega)$, while have large differences in some of $B \rightarrow b_1 \rho(\omega)$ decays.

- For the decays $B^- \rightarrow a_1^0 \rho^-, b_1^0 \rho^-$ and $\bar{B}^0 \rightarrow b_1^0 \rho^0, b_1^0 \omega$, where the transverse polarization fractions range from 4.7 to 7.5%, we calculate their direct CP-violating asymmetries with neglecting the transverse polarizations and find that those for two charged decays have smaller values, which are about 11.8% and -3.7% , respectively.

Acknowledgment

This work is partly supported by the National Natural Science Foundation of China under Grant No. 11147004, and by Foundation of Henan University of Technology under Grant No. 2009BS038. The author would like to thank Cai-Dian Lü and Wei Wang for helpful discussions.

-
- [1] Y. Li, C. D. Lu, Phys. Rev. D **73**, 014024 (2006).
 - [2] H. W. Huang, *et al.*, Phys. Rev. D **73**, 014011 (2006).
 - [3] A. Ali, *et al.*, Phys. Rev. D **76**, 074018 (2007).
 - [4] M. Beneke, J. Rohrer, D.S. Yang, Phys. Lett. B **768**, 51 (2007).
 - [5] K. Abe, *et al.*, [Belle Collaboration], Phys. Rev. Lett. **87**, 161601 (2001).
 - [6] B. Aubert, *et al.*, [BABAR Collaboration], arXiv:hep-ex/0207085 (2002).
 - [7] B. Aubert, *et al.*, [BABAR Collaboration], Phys. Rev. D **74**, 031104 (2006), arXiv:hep-ex/0605024.
 - [8] B. Aubert, *et al.*, [BABAR Collaboration], Phys. Rev. D **80**, 051101 (2009), arXiv:hep-ex/0907.3485v1.
 - [9] H. Y. Cheng, K. C. Yang, Phys. Rev. D **78**, 094001 (2008).
 - [10] G. Calderon, J.H. Munoz and C.E. Vera, Phys. Rev. D **76**, 094019 (2007), arXiv:hep-ph/0705.1181.
 - [11] C.D. Lu and M.Z. Yang, Eur. Phys. J. C **28**, 515 (2003).
 - [12] R. H. Li, C. D. Lu, W. Wang, Phys. Rev. D **79**, 034014 (2009).
 - [13] C. Amsler, *et al.*, [Particle Data Group], Phys. Lett. B **667**, 1 (2008).
 - [14] P. Ball, G. W. Jones and R. Zwicky, Phys. Rev. D **75**, 054004 (2007).
 - [15] P. Ball and R. Zwicky, JHEP **0604**, 046 (2006).
 - [16] P. Ball and G. W. Jones, JHEP **0703**, 069 (2007).
 - [17] K. C. Yang, JHEP **0510**, 108 (2005).
 - [18] K. C. Yang, Nucl. Phys. B **776**, 187 (2007).
 - [19] C. D. Lu, K. Ukai, and M. Z. Yang, Phys. Rev. D **63**, 074009 (2001).
 - [20] Y. Y. Keum, H. n. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).
 - [21] S. Mishima, Phys. Lett. B **521**, 252 (2001); C. H. Chen, Y. Y. Keum, and H. n. Li, Phys. Rev. D **64**, 112002 (2001).
 - [22] H. n. Li and B. Tseng, Phys. Rev. D **57**, 443 (1998).

- [23] H. n. Li, Phys. Rev. D **66**, 094010 (2002).
- [24] H. n. Li and K. Ukai, Phys. Lett. B **555**, 197 (2003).
- [25] H. n. Li and H. L. Yu, Phys. Rev. D **53**, 2480 (1996).
- [26] K. Nakamura, *et al.*, [Particle Data Group], J. Phys. **G37**, 481 (2010).
- [27] CKMfitter Group, <http://ckmfitter.in2p3.fr>.
- [28] Y. Li, C. D. Lu, W. Wang, Phys. Rev. D **80**, 014024 (2009).
- [29] Z. Q. Zhang, H. F. Ou, L. X. Lu, J. Phys. **G38**, 095005 (2011).